## [Ice Cream Sundae clipart](http://www.clipartguide.com/_pages/1386-0905-2902-0116.html)

## Culminating Task: Growing by Leaps and Bounds

**Mathematical Goals**

* Create one-variable exponential equations from contextual situations.
* Use properties of exponents to solve and interpret the solution to exponential equations in context.
* Write and graph an equation to represent an exponential relationship.
* Graph equations on coordinate axes with labels and scales.
* Use technology to explore exponential graphs.

**Common Core State Standards**

**MCC9-12.A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and ~~quadratic functions~~, ~~and simple rational and~~ exponential functions.

**MCC9-12.A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

**MCC9-12.A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

**MCC9-12.A.CED.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations*.*

**MCC9-12.N.Q.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

**MCC9-12.N.Q.2** Define appropriate quantities for the purpose of descriptive modeling.

**MCC9-12.N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

**MCC9-12.A.SSE.1** Interpret expressions that represent a quantity in terms of its context.

**MCC9-12.A.SSE.1a** Interpret parts of an expression, such as terms, factors, and coefficients.

**MCC9-12.A.SSE.1b** Interpret complicated expressions by viewing one or more of their parts as a single entity.

**Standards for Mathematical Practice**

**1. Make sense of problems and persevere in solving them.**

**2. Reason abstractly and quantitatively.**

**3. Construct viable arguments and critique the reasoning of others.**

**4. Model with mathematics.**

**5. Use appropriate tools strategically.**

**6. Attend to precision.**

**7. Look for and make use of structure.**

**8. Look for and express regularity in repeated reasoning.**

**Part 1: Meet Linda**

Linda’s lifelong dream has been to open her own business. After working, sacrificing, and saving, she finally has enough money to open up an ice cream business. The grand opening of her business is scheduled for the Friday of Memorial Day weekend. She would like to have a soft opening for her business on the Tuesday before. The soft opening should give her a good idea of any supply or personnel issues and give her time to correct them before the big official opening.

A soft opening means that the opening of the business is not officially announced; news of its opening is just spread by word of mouth (see, not all rumors are bad!). Linda needs a good idea of when she should begin the rumor in order for it to spread reasonably well before her soft opening. She has been told that about 10% of the people who know about an event will actually attend it. Based on this assumption, if she wants to have about 50 people visit her store on the Tuesday of the soft opening, she will need 500 people to know about it.

1. Linda plans to tell one person each day and will ask that person to tell one other person each day through the day of the opening, and so on. Assume that each new person who hears about the soft opening is also asked to tell one other person each day through the day of the opening and that each one starts the process of telling their friends on the day after he or she first hears. When should Linda begin telling others about the soft opening in order to have at least 500 people know about it by the day it occurs?
2. Let *x* represent the day number and let *y* be the number of people who know about the soft opening on day *x*. Consider the day before Linda told anyone to be Day 0, so that Linda is the only person who knows about the opening on Day 0. Day 1 is the first day that Linda told someone else about the opening.
   1. Complete the following table.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Day | 0 | 1 | 2 | 3 | 4 | 5 |
| Number of people who know | 1 | 2 |  |  |  |  |

* 1. Graph the points from the table in part a.

1. Write an equation that describes the relationship between *x* (day) and *y* (number of people who know) for the situation of spreading the news about the soft opening of Linda’s ice cream store.
2. Does your equation describe the relationship between day and number who know about Linda’s ice cream store soft opening completely? Why or why not?

**Part 2: What if?**

The spread of a rumor or the spread of a disease can be modeled by a type of function known as exponential function; in particular, an exponential ***growth*** function. An **exponential function** has the form

,

where *a* is a non-zero real number and *b* is a positive real number other than 1. An exponential growth function has a value of b that is greater than 1.

1. In the case of Linda’s ice cream store, what values of *a* and *b* yield an exponential function to model the spread of the rumor of the soft store opening?
2. In this particular case, what is an appropriate domain for the exponential function? What range corresponds to this domain?
3. In part 1, item 2, you drew a portion of the graph of this function. Does it make sense to connect the dots on the graph? Why or why not?
4. How would the graph change if Linda had told two people each day rather than one and had asked that each person also tell two other people each day?
5. How would the equation change if Linda had told two people each day rather than one and had asked that each person also tell two other people each day? What would be the values of *a* and *b* in this case?
6. How long would it take for at least 500 people to find out about the opening if the rumor spread at this new rate?

**Part 3: The Beginning of a Business**

How in the world did Linda ever save enough to buy the franchise to an ice cream store? Her mom used to say, “That Linda, why she could squeeze a quarter out of a nickel!” The truth is that Linda learned early in life that patience with money is a good thing. When she was just about 9 years old, she asked her dad if she could put her money in the bank. He took her to the bank and she opened her very first savings account.

Each year until Linda was 16, she deposited her birthday money into her savings account. Her grandparents (both sets) and her parents each gave her money for her birthday that was equal to twice her age; so on her ninth birthday, she deposited $54 ($18 from each couple).

Linda’s bank paid her 3% interest, compounded quarterly. The bank calculated her interest using the following standard formula:



where *A* = final amount, *P* = principal amount, *r* = interest rate, *n* = number of times per year the interest is compounded, and *t* is the number of years the money is left in the account.

1. Verify the first entry in the following chart, and then complete the chart to calculate how much money Linda had on her 16th birthday. Do not round answers until the end of the computation, then give the final amount rounded to the nearest cent.

|  |  |  |  |
| --- | --- | --- | --- |
| Age | Birthday $ | Amt from previous year plus Birthday | Total at year end |
| 9 | 54 | 0 | 55.63831630 |
| 10 |  |  |  |
| 11 |  |  |  |
| 12 |  |  |  |
| **.**  **.**  **.** | **.**  **.**  **.** | **.**  **.**  **.** | **.**  **.**  **.** |

1. On her 16th birthday, the budding entrepreneur asked her parents if she could invest in the stock market. She studied the newspaper, talked to her economics teacher, researched a few companies and finally settled on the stock she wanted. She invested all of her money in the stock and promptly forgot about it. When she graduated from college on her 22nd birthday, she received a statement from her stocks and realized that her stock had appreciated an average of 10% per year. How much was her stock worth on her 22nd birthday?
2. When Linda graduated from college, she received an academic award that carried a $500 cash award. On her 22nd birthday, she used the money to purchase additional stock. She started her first job immediately after graduation and decided to save $50 each month. On her 23rd birthday she used the $600 (total of her monthly amount) savings to purchase new stock. Each year thereafter she increased her total of her savings by $100 and, on her birthday each year, used her savings to purchase additional stock. Linda continued to learn about stocks and managed her accounts carefully. On her 35th birthday she looked back and saw that her stock had appreciated at 11% during the first year after college and that the rate of appreciation increased by 0.25% each year thereafter. At age 34, she cashed in enough stock to make a down payment on a bank loan to purchase her business. What was her stock worth on her 34th birthday? Use a table like the one below to organize your calculations.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Age | Amt from previous year | Amt Linda added from savings that year | Amount invested for the year | Interest rate for the year | Amt at year end |
| 22 | 998.01 | 500 | 1498.01 | 11.00% | 1662.79 |
| 23 | 1662.79 | 600 |  | 11.25% |  |
| 24 |  | 700 |  | 11.50% |  |
| 25 |  | 800 |  | 11.75% |  |
| **.**  **.**  **.** | **.**  **.**  **.** | **.**  **.**  **.** | **.**  **.**  **.** | **.**  **.**  **.** | **.**  **.**  **.** |